

Non-Gaussianity in the modulated reheating scenario

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(Dated: February 1, 2008)

Abstract

We investigate the non-Gaussianity of primordial curvature perturbations in the modulated reheating scenario where the primordial perturbation is generated due to the spatial fluctuation of the rate of the inflaton decay to radiation. We use the δN formalism to evaluate the trispectrum of the curvature perturbation as well as its bispectrum. We give expressions for three non-linear parameters f_{NL} , τ_{NL} and g_{NL} in the modulated reheating scenario. If both the intrinsic non-Gaussianity of scalar field fluctuations and third the derivative of the decay rate with respect to the scalar fields are negligibly small, g_{NL} has at least the same order of magnitude as f_{NL} . We also give a general inequality between f_{NL} and τ_{NL} , which is true for other inflationary scenarios as long as the primordial non-Gaussianity comes from super-horizon evolution.

PACS numbers: 98.80.Cq

I. INTRODUCTION

Recent observations of cosmic microwave background (CMB) anisotropies give strong evidence that primordial density fluctuations are almost Gaussian, scale-invariant, and adiabatic [1]. During inflation, they are generated as vacuum fluctuations of light fields and are stretched to cosmological scales to explain the large scale structure of the universe [2]. Such a light field responsible for density fluctuations has been considered to be the inflaton itself for a long time.

Recently, alternative candidates for such a light field have been proposed. One attractive example is the curvaton [3, 4], which is effectively massless and acquires fluctuations during inflation. After inflation, it becomes effectively massive and contributes to a non-negligible fraction of the energy density of the universe. Then, after it decays, eventually density fluctuations induced by the curvaton are converted to adiabatic ones and can dominate over those generated by the inflaton itself. Another interesting candidate is a light field whose expectation value determines the coupling constant of the inflaton to standard model particles [5]. Such a light field will fluctuate during inflation, which leads to fluctuation of the decay rate of the inflaton. Then, this latter spatial fluctuation induces that of the reheating temperature, which eventually generates curvature perturbations. Some variants of these two alternative candidates have also been considered related to the fluctuations of masses and annihilation cross sections [6], and the preheating mechanism [7].

In order to determine which light field is actually responsible for primordial density fluctuations, the deviation from Gaussianity of the curvature perturbations is of great use. If density fluctuations are completely Gaussian, their bispectra, characterized by a parameter f_{NL} , and (connected) trispectra, characterized by parameters g_{NL} and τ_{NL} vanish. Therefore, estimation of the bispectra and trispectra is important to identify the light field. In [8], it was shown that the bispectrum is significantly suppressed by the slow-roll parameters up to an undetectable level in a single field slow-roll inflation model.¹ After [8], the trispectrum, as well as bispectrum, were evaluated in a multi-field configuration and a curvaton scenario. Though spectra are still suppressed by slow-roll parameters in a multi-field configuration [10, 11], they can be significantly large in the curvaton scenario [12, 13]. On the contrary, only the bispectrum is calculated in the modulated reheating scenario [14, 15, 16]. Since the bispectrum can be large in both the curvaton scenario and the modulated reheating scenario, trispectrum may be useful to discriminate between them. Though the present constraint on the trispectrum is not particularly severe, and is roughly given by $|\tau_{NL}| \lesssim 10^8$ [17], a value of $|\tau_{NL}| \sim 560$ will be detectable by the Planck satellite [18].²

The main purpose of our paper is to estimate the trispectrum in the modulated reheating scenario. We use the δN formalism, which is a powerful approach to evaluate the non-Gaussianity of curvature perturbations simply because it requires a homogeneous background solution [20, 21, 22, 23]. Then, we use the same formalism to evaluate the trispectrum of curvature perturbations as well as its bispectrum.

This paper is organized as follows. In the next section, we give a brief review of the δN

¹ It has recently pointed out that there is a possibility that an all-sky 21-cm experiment is sensitive to a value of $|f_{NL}| \sim 0.01$ [9].

² In order to obtain these constraints, one must compute not only the non-Gaussianities of primordial fluctuations but also other non-linear effects entering in the CMB such the non-linear evolution of the perturbations after inflation [19].

formalism and a definition of the three non-linear parameters f_{NL} , τ_{NL} and g_{NL} given in [13, 17, 24]. We also give a general inequality between f_{NL} and τ_{NL} which is not found in the literature. In Sec. III, we study the background dynamics in the modulated reheating scenario. In Sec. IV, we study the perturbations in this scenario and give expressions for f_{NL} , τ_{NL} and g_{NL} . The final section is devoted to a summary. We use the units $8\pi G = 1$.

II. δN FORMALISM

According to the δN formalism [20, 21, 22, 23], the curvature perturbation on a uniform energy density hypersurface ζ at time t_f is, on sufficiently large scales, equal to the perturbation in the time integral of the local expansion from an initial flat hypersurface ($t = t_i$) to the final uniform energy density hypersurface. On sufficiently large scales, the local expansion can be approximated quite well by the expansion of the unperturbed Friedmann universe. Hence

$$\zeta(t_f, \vec{x}) = N(t_i, t_f, \vec{x}) - (\text{spatial average}), \quad (1)$$

where the e -folding number $N(t_i, t_f, \vec{x})$ is defined by the time integral of the local Hubble parameter,

$$N(t_i, t_f, \vec{x}) = \int_{t_i}^{t_f} H(t, \vec{x}) dt. \quad (2)$$

In many inflationary scenarios, which include the modulated reheating scenario, the dynamics of the universe between t_i and t_f is determined by the values of the relevant scalar fields ϕ^I at t_i and by the e -folding number which becomes a function of $\phi^I(t_i, \vec{x})$. Hence the curvature perturbation at t_f is given by

$$\zeta(t_f, \vec{x}) \approx N_I \delta\phi^I + \frac{1}{2} N_{IJ} \delta\phi^I \delta\phi^J + \dots - (\text{spatial average}), \quad (3)$$

where $\delta\phi^I$ is the perturbation of ϕ^I on the flat hypersurface at t_i , and N_I , N_{IJ} , \dots are given by

$$N_I = \frac{\partial N}{\partial \phi^I}, \quad N_{IJ} = \frac{\partial^2 N}{\partial \phi^I \partial \phi^J}, \quad \dots \quad (4)$$

Because solutions of the unperturbed Friedmann equation give N_I, N_{IJ}, \dots , the knowledge of the background solutions is enough to know the higher order correlation functions of ζ .

The connected parts of the power spectrum, bispectrum and trispectrum are defined as

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle_c = (2\pi)^3 P_\zeta(k_1) \delta(\vec{k}_1 + \vec{k}_2), \quad (5)$$

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle_c = (2\pi)^3 B_\zeta(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3), \quad (6)$$

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle_c = (2\pi)^3 T_\zeta(k_1, k_2, k_3, k_4) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4). \quad (7)$$

Here $\langle \dots \rangle_c$ means that we take the connected part of $\langle \dots \rangle$. Using Eq. (3), we can express P_ζ , B_ζ and T_ζ in terms of the correlation functions of $\delta\phi^I$, which are given to leading order

by [13],

$$P_\zeta(k) = N_I N_J P^{IJ}(k), \quad (8)$$

$$B_\zeta(k_1, k_2, k_3) = N_I N_J N_K B^{IJK}(k_1, k_2, k_3) + N_I N_{JK} N_K (P^{IK}(k_1) P^{JL}(k_2) + 2 \text{ perms}) \quad (9)$$

$$\begin{aligned} T_\zeta(k_1, k_2, k_3, k_4) = & N_I N_J N_K N_L T^{IJKL}(k_1, k_2, k_3, k_4) \\ & + N_{IJ} N_K N_L N_M (P^{IK}(k_1) B^{JLM}(k_{12}, k_3, k_4) + 11 \text{ perms.}) \\ & + N_{IJ} N_{KL} N_M N_N (P^{JL}(k_{13}) P^{IM}(k_3) P^{JN}(k_4) + 11 \text{ perms.}) \\ & + N_{IJK} N_L N_M N_N (P^{IL}(k_2) P^{JM}(k_3) P^{KN}(k_4) + 3 \text{ perms.}), \end{aligned} \quad (10)$$

where $k_{ij} = |\vec{k}_i - \vec{k}_j|$ and P^{IJ} , B^{IJK} , T^{IJKL} are the power spectrum, bispectrum and trispectrum of the scalar fields respectively, defined by

$$\langle \delta\phi_{\vec{k}_1}^I \delta\phi_{\vec{k}_2}^J \rangle_c = (2\pi)^3 P^{IJ}(k_1) \delta(\vec{k}_1 + \vec{k}_2), \quad (11)$$

$$\langle \delta\phi_{\vec{k}_1}^I \delta\phi_{\vec{k}_2}^J \delta\phi_{\vec{k}_3}^K \rangle_c = (2\pi)^3 B^{IJK}(k_1, k_2, k_3) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3), \quad (12)$$

$$\langle \delta\phi_{\vec{k}_1}^I \delta\phi_{\vec{k}_2}^J \delta\phi_{\vec{k}_3}^K \delta\phi_{\vec{k}_4}^L \rangle_c = (2\pi)^3 T^{IJKL}(k_1, k_2, k_3, k_4) \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4). \quad (13)$$

If $\delta\phi^I$ are independent Gaussian variables with the same variance which we denote as P , then $P^{IJ} = P\delta^{IJ}$ and B^{IJK} and T^{IJKL} vanish. In such a case, deviation from Gaussianity of the primordial perturbation comes only from super-horizon evolution, and the non-Gaussianity is characterized by three constant parameters f_{NL} , τ_{NL} and g_{NL} defined by

$$B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{NL} (P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perms.}), \quad (14)$$

$$\begin{aligned} T_\zeta(k_1, k_2, k_3, k_4) = & \tau_{NL} (P_\zeta(k_{13}) P_\zeta(k_3) P_\zeta(k_4) + 11 \text{ perms.}) \\ & + \frac{54}{25} g_{NL} (P_\zeta(k_2) P_\zeta(k_3) P_\zeta(k_4) + 3 \text{ perms.}). \end{aligned} \quad (15)$$

Using Eqs. (8)-(10), we have the following expressions,

$$f_{NL} = \frac{5 N_I N_J N^{IJ}}{6 (N_K N^K)^2}, \quad (16)$$

$$\tau_{NL} = \frac{N_{IJ} N^{IK} N^J N_K}{(N_L N^L)^3}, \quad (17)$$

$$g_{NL} = \frac{25 N_{IJK} N^I N^J N^K}{54 (N_L N^L)^3}. \quad (18)$$

Eq. (16) has been given in [24]. Eq. (17) has been given in the arXiv version of [17] and also in [13]. Eq. (18) has been given in [13].

Here we provide a relation between f_{NL} and τ_{NL} which is not found in the literature. From the Cauchy-Schwarz inequality, we have the following

$$\tau_{NL} \geq \frac{36}{25} f_{NL}^2. \quad (19)$$

We have equality if and only if the vector N_I is an eigenvector of the matrix N_{IJ} . The single inflation model yields $\tau_{NL} = \frac{36}{25} f_{NL}^2$. However in multi-field inflation, there is a possibility that two vectors N_I and $N_{IJ} N^J$ are nearly orthogonal. In such a case, f_{NL} is very small but τ_{NL} and possibly also g_{NL} remains finite. Hence the leading non-Gaussianity comes not from the bispectrum but from the trispectrum.

III. BACKGROUND DYNAMICS OF THE MODULATED REHEATING SCENARIO

In the modulated reheating scenario [5], the decay rate Γ of the inflaton S is a function of scalar fields ϕ^I (not necessarily a single field) which are light during inflation. We assume that fluctuation of the inflaton field generates negligible curvature perturbation. Then the detailed form of the inflaton potential $U(S)$ during inflation is not important for the scenario. We only require that $U(S)$ around the minimum is approximated well by a term quadratic in S . After inflation, the inflaton oscillates around the minimum of the potential. The energy density of the inflaton ρ_S averaged over one period of the oscillation behaves as a function of the e -folding number $\propto e^{-3N}$. Hence we regard ρ_S as dust. The inflaton decays into radiation with rate Γ which depends on the expectation values of ϕ^I . Then the background equations are given by

$$\frac{d\rho_S}{dN} + 3\rho_S = -\frac{\Gamma}{H}\rho_S, \quad (20)$$

$$\frac{d\rho_r}{dN} + 4\rho_r = \frac{\Gamma}{H}\rho_S, \quad (21)$$

$$H^2 = \frac{1}{3}(\rho_S + \rho_r), \quad (22)$$

where ρ_r is the energy density of radiation. Spatial fluctuations of these light fields induce the fluctuation of the decay rate and the curvature perturbation. By solving the above equations from the end of inflation to the completion of reheating with the initial conditions $\rho_S(0) = \rho_0 = 3H_0^2$, $\rho_r(0) = 0$, we can obtain a relation between N and Γ . Until the Hubble function drops to H_f ($H_f \ll \Gamma$), the e -folding number N can be written formally as

$$N = \frac{1}{2} \log \frac{H_0}{H_f} + Q\left(\frac{\Gamma}{H_0}\right), \quad (23)$$

where

$$\exp\left[4Q(\Gamma/H_0)\right] \equiv \int_0^\infty dN' \frac{\Gamma}{H(N)} e^{4N'} \frac{\rho_S(N')}{\rho_0}. \quad (24)$$

For the two limiting cases, we have the approximate form of $Q(x)$ as

$$Q(x) = \frac{1}{4x} + \mathcal{O}(x^{-2}) \quad x \gg 1, \quad (25)$$

$$Q(x) = -\frac{1}{6} \log x + \mathcal{O}(x) \quad x \ll 1. \quad (26)$$

For arbitrary x , we do not have an analytic form for $Q(x)$ and we have to solve the background equations numerically. We show $Q(x)$ calculated numerically in Fig. 1. We also find an accurate fitting formula for $Q(x)$ of the form,

$$Q_{\text{fit}}(x) = \frac{1}{4} \frac{r(x) + 2}{r(x) + 3} \log\left(1 + \frac{1}{x}\right). \quad (27)$$

For $r(x) = 2.16x^{0.72}$, the relative error is within 2 percent. For $r(x) = 1.7x^{0.9} + 0.3x^{0.18}$, the relative error is within 0.5 percent.

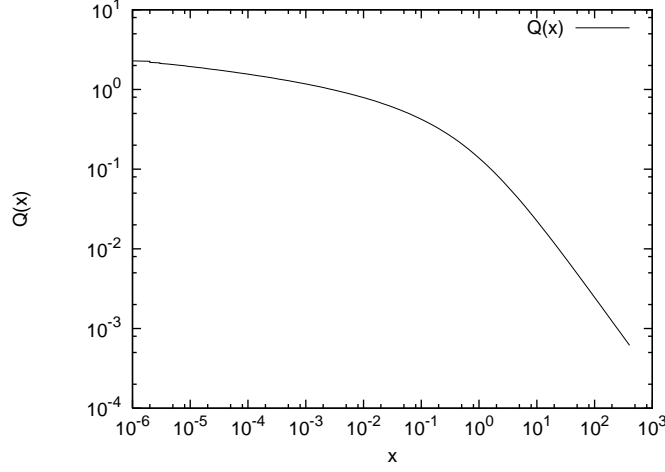


FIG. 1: Plot of $Q(x)$ calculated numerically. The fitting formula $Q_{\text{fit}}(x)$ lies within the solid line.

IV. PERTURBATION IN THE MODULATED REHEATING SCENARIO

From Eq. (23), we can calculate δN in terms of $\delta\Gamma$,

$$\delta N = xQ'(x)\frac{\delta\Gamma}{\Gamma} + \frac{x^2}{2}Q''(x)\left(\frac{\delta\Gamma}{\Gamma}\right)^2 + \frac{x^3}{6}Q'''(x)\left(\frac{\delta\Gamma}{\Gamma}\right)^3 + \dots \quad (28)$$

In the modulated reheating scenario, the decay rate depends on the scalar fields which are almost massless during inflation. Hence the perturbation of the decay rate can be written as a function of the perturbation of the scalar fields,

$$\delta\Gamma = \Gamma_I\delta\phi^I + \frac{1}{2}\Gamma_{IJ}\delta\phi^I\delta\phi^J + \frac{1}{6}\Gamma_{IJK}\delta\phi^I\delta\phi^J\delta\phi^K + \dots \quad (29)$$

From these equations, N_I , N_{IJ} , N_{IJK} can be written as

$$N_I = xQ'(x)\frac{\Gamma_I}{\Gamma}, \quad (30)$$

$$N_{IJ} = xQ'(x)\frac{\Gamma_{IJ}}{\Gamma} + x^2Q''(x)\frac{\Gamma_I}{\Gamma}\frac{\Gamma_J}{\Gamma}, \quad (31)$$

$$N_{IJK} = xQ'(x)\frac{\Gamma_{IJK}}{\Gamma} + x^2Q''(x)\left(\frac{\Gamma_I}{\Gamma}\frac{\Gamma_{JK}}{\Gamma} + \frac{\Gamma_J}{\Gamma}\frac{\Gamma_{KI}}{\Gamma} + \frac{\Gamma_K}{\Gamma}\frac{\Gamma_{IJ}}{\Gamma}\right) + x^3Q'''(x)\frac{\Gamma_I}{\Gamma}\frac{\Gamma_J}{\Gamma}\frac{\Gamma_K}{\Gamma} \quad (32)$$

Substituting these latter into Eqs. (8)-(10) yields the power spectrum, bispectrum and trispectrum of the primordial curvature perturbation in the modulated reheating scenario. Note that Eqs. (30)-(32) are only correct if t_i is taken as the time when inflation ends. Hence in Eqs. (8)-(10), we must use the power spectrum, bispectrum and trispectrum of scalar field fluctuation at the end of inflation.

Zaldarriaga [14] evaluated the bispectrum B of the curvature perturbations at reheating by taking into account not only the nonlinear evolution of the scalar fluctuations but also their intrinsic non-Gaussianities [14]. In particular, in order to estimate the intrinsic non-Gaussianity of a scalar field, he calculates the leading order quantum three-point correlation

function with the assumption that Γ depends only on a single field ϕ ,

$$\langle \hat{\phi}_{\vec{k}_1}(\eta) \hat{\phi}_{\vec{k}_2}(\eta) \hat{\phi}_{\vec{k}_3}(\eta) \rangle_c = -i \langle 0 | \int_{-\infty}^{\eta} d\eta' [\hat{\phi}_{\vec{k}_1}(\eta) \hat{\phi}_{\vec{k}_2}(\eta) \hat{\phi}_{\vec{k}_3}(\eta), \hat{V}(\eta')] 0 \rangle, \quad (33)$$

where $\eta \equiv \int_{-\infty}^t \frac{dt}{a}$ is the conformal time. In Ref. [14], mode functions in pure de-Sitter spacetime are used throughout the evolution of the above correlation function, though the use of such mode functions are not necessarily justified during the oscillatory phase after inflation. In our formalism, we have only to know the field fluctuations at the end of inflation. However, even in this case, mode functions in pure de-Sitter spacetime are not good approximations after the relevant scale crosses the horizon. This is because the Hubble parameter may decrease by an order of magnitude from the horizon exit to the end of inflation. In this paper, instead of using Eq. (33) until the end of inflation, we use it only until the relevant mode exits the horizon. After the horizon crossing, we evolve $\delta\phi^I$ by perturbing the classical unperturbed equations, in keeping with the spirit of the δN formalism, which correctly takes into account the evolution of the Hubble parameter.

We start from the evolution of scalar field fluctuations on super-horizon scales, which can be described well by the classical treatment. Then, the background equations are given by

$$\frac{d^2\phi^I}{dN^2} + \left(3 + \frac{1}{H} \frac{dH}{dN}\right) \frac{d\phi^I}{dN} + \frac{V^I}{H^2} = 0. \quad (34)$$

We assume that scalar fields slow-roll during the whole epochs of interest, which enables us to approximate the background equations as

$$\frac{d\phi^I}{dN} \simeq -\frac{1}{3 + \frac{H'}{H}} \frac{V^I}{H^2}. \quad (35)$$

Throughout this paper, it is also assumed that the total energy density of the universe is dominated by the inflaton potential, and the dependence of ϕ^I on H is negligible.

Let N_* be the e -folding slightly after the horizon crossing and N_f be the e -folding at the end of inflation. Slightly after the horizon crossing, the scalar field fluctuations turn into classical variables with their magnitude given by $\delta\phi_*^I$. Then $\delta\phi^I(N)$ after N_* is a function of $\delta\phi_*^I$, which can be Taylor-expanded with respect to $\delta\phi_*^I$. For the sake of the evaluation of the leading bispectrum and trispectrum of scalar fields at $N = N_f$, it is enough to expand $\delta\phi^I(N)$ to third order in $\delta\phi_*^I$. Up to third order, Eq. (35) yields

$$\phi^I(N_f) = \Lambda_J^I(N_f, N_*) \delta\phi_*^J + \frac{1}{2} \Theta_{JK}^I(N_f, N_*) \delta\phi_*^J \delta\phi_*^K + \frac{1}{6} \Xi_{JKL}^I(N_f, N_*) \delta\phi_*^J \delta\phi_*^K \delta\phi_*^L, \quad (36)$$

where

$$\Lambda_J^I(N, N') \equiv \left[T \exp \left(\int_{N'}^N dN'' P(N'') \right) \right]_J^I, \quad (37)$$

$$\Theta_{JK}^I(N_f, N_*) = \int_{N_*}^{N_f} dN' \Lambda_L^I(N_f, N') Q_{MN}^L(N') \Lambda_J^M(N', N_*) \Lambda_K^N(N', N_*), \quad (38)$$

$$\Xi_{JKL}^I(N_f, N_*) = \frac{3}{2} \int_{N_*}^{N_f} dN' \Lambda_{I'}^I(N, N') Q_{MN}^{I'}(N') \Lambda_J^M(N', N_*)$$

$$\begin{aligned}
& \times \int_{N_*}^{N'} dN'' \Lambda_{N'}^N(N', N'') Q_{PQ}^{N'}(N'') \Lambda_K^P(N'', N_*) \Lambda_L^Q(N'', N_*) \\
& + \int_{N_*}^N dN' \Lambda_{I'}^I(N, N') R_{J'K'L'}^{I'}(N') \Lambda_J^{J'}(N', N_*) \Lambda_K^{K'}(N', N_*) \Lambda_L^{L'}(N', N_*)
\end{aligned} \tag{39}$$

$$P_J^I = -\frac{1}{3 + \frac{H'}{H}} \frac{V_J^I}{H^2}, \tag{40}$$

$$Q_{JK}^I = -\frac{1}{3 + \frac{H'}{H}} \frac{V_{JK}^I}{H^2}, \tag{41}$$

$$R_{JKL}^I = -\frac{1}{3 + \frac{H'}{H}} \frac{V_{JKL}^I}{H^2}. \tag{42}$$

A. Power spectrum of $\delta\phi^I$ at the end of inflation

To leading order, the two-point function of $\delta\phi^I$ at the end of inflation is given by

$$P^{IJ}(N_f) = \Lambda_K^I(N_f, N_*) \Lambda_L^J(N_f, N_*) P_*^{KL}(k_1). \tag{43}$$

Here, slightly after the horizon crossing, P_*^{IJ} is given to leading order by,

$$P_*^{IJ} = \frac{(2\pi)^3}{4\pi k^3} \left(\frac{H_*}{2\pi} \right)^2 \delta^{IJ} \equiv P_* \delta^{IJ}. \tag{44}$$

Any corrections to Eq. (44) are suppressed by the slow-roll parameters [13]. Only fluctuations that extend to super-horizon scales at the end inflation contribute to the fluctuation of Γ . Such fields must be massless during inflation and we assume $\Lambda_J^I \approx \delta_J^I$, which yields the corresponding power spectrum as

$$P^{IJ}(N_f) \approx P_* \delta^{IJ}. \tag{45}$$

B. Bispectrum of $\delta\phi^I$ at the end of inflation

To leading order, the bispectrum of $\delta\phi^I$ at the end of inflation is given by

$$B^{IJK}(k_1, k_2, k_3)(N_f) \simeq B_*^{IJK}(k_1, k_2, k_3) + \Theta^{IJK}(N_f, N_*) (P_*(k_1)P_*(k_2) + 2 \text{ perms.}). \tag{46}$$

B_*^{IJK} , which is the bispectrum slightly after the horizon crossing, can be evaluated by quantum perturbation theory [14]

$$B_*^{IJK}(k_1, k_2, k_3) = -2V_{IJK} \text{Re} \left[i g_{k_1}(\eta_*) g_{k_2}(\eta_*) g_{k_3}(\eta_*) \int_{-\infty}^{\eta_*} d\eta a^4(\eta) g_{k_1}^*(\eta) g_{k_2}^*(\eta) g_{k_3}^*(\eta) \right], \tag{47}$$

where $g_k(\eta)$ is the mode function in de-Sitter spacetime and is given by,

$$g_k(\eta) = \frac{iH}{\sqrt{2k^{3/2}}} (1 + ik\eta) e^{-ik\eta}. \tag{48}$$

Soon after the horizon crossing, $k_i\eta$ becomes smaller than unity. In such a phase, the leading bispectrum of Eq. (47) is given by

$$B_*^{IJK}(k_1, k_2, k_3) = -\frac{H_*^2}{4} \frac{k_t^3}{(k_1 k_2 k_3)^3} V_{IJK} \times \left[\left(\gamma + \frac{1}{2} \log(k_t \eta_*)^2 \right) \left(-\frac{1}{3} + \frac{\sum_{i<j} k_i k_j}{k_t^2} - \frac{k_1 k_2 k_3}{k_t^3} \right) + \frac{4}{9} - \frac{\sum_{i<j} k_i k_j}{k_t^2} \right] \quad (49)$$

where $\gamma = 0.577\dots$ is Euler's constant.³ Because the second line in Eq. (49) is $\mathcal{O}(1)$, $B_* = \mathcal{O}(V_{IJK}/H_*^4)$. On the other hand, the bispectrum of the second term in Eq. (46) can be estimated as

$$\Theta^{IJK}(N_f, N_*) P_*(k_1) P_*(k_2) = \mathcal{O} \left(\frac{1}{H_*^2} \int_{N_*}^{N_f} dN \frac{V_{IJK}}{H^2(N)} \right), \quad (50)$$

which is expected to be larger than B_* because $N_f - N_*$ is typically $\mathcal{O}(10)$. Hence the leading bispectrum of $\delta\phi^I$ at the end of inflation comes from super-horizon evolution,

$$B^{IJK}(N_f) \approx \Theta^{IJK}(N_f, N_*) (P_*(k_1) P_*(k_2) + 2 \text{ perms.}) . \quad (51)$$

Thus, the corresponding non-linear parameter f_{NL} becomes⁴

$$\frac{6}{5} f_{NL} = \frac{1}{x Q'(x)} \frac{\Gamma \Gamma_I \tilde{\Theta}^I}{\Gamma_K \Gamma^K} + \frac{1}{x Q'(x)} \frac{\Gamma \Gamma_I \tilde{\Gamma}_{(2)}^I}{(\Gamma_K \Gamma^K)^{3/2}} + \frac{Q''(x)}{Q'^2(x)}, \quad (52)$$

where $\tilde{\Theta}^I$ and $\tilde{\Gamma}_{(2)}^I$ are projected vectors of Θ^{IJK} and Γ^{IJ} , respectively,

$$\tilde{\Theta}^I = \frac{\Gamma_J \Gamma_K}{\Gamma_L \Gamma^L} \Theta^{IJK}, \quad (53)$$

$$\tilde{\Gamma}_{(2)}^I = \frac{\Gamma_J \Gamma^{IJ}}{(\Gamma_K \Gamma^K)^{1/2}}. \quad (54)$$

One should notice that the first term in Eq. (52) represents intrinsic non-Gaussianity due to cubic interactions of the scalar fields. The second term comes from non-linearity between Γ and ϕ^I . The third term comes from non-linearity between ζ and $\delta\Gamma$, which only depends on Γ through the argument $x = \Gamma/H_0$ of $Q(x)$. $Q''(x)/Q'^2(x)$ takes minimum value 6 at $x = 0$ and becomes larger for $x > 1$ (see Fig. 2). Hence if both the intrinsic non-Gaussianity of the scalar field fluctuations and the non-linearity between Γ and ϕ^I are negligibly small, then $f_{NL} > 5$.

³ This expression for the bispectrum is slightly different from that given in Ref. [14], though the difference is not essential. Thanks to a second look by the author of Ref. [14], we reach the same result, which is given above.

⁴ Note that the definition of f_{NL} here is different in sign from that in [16]

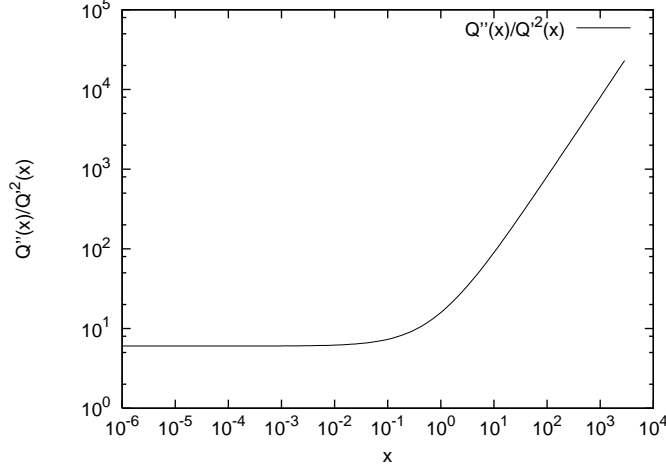


FIG. 2: Plot of $Q''(x)/Q'^2(x)$.

C. Trispectrum of $\delta\phi^I$ at the end of inflation

To leading order, the trispectrum of $\delta\phi^I$ at the end of inflation is given by

$$\begin{aligned}
T^{IJKL}(k_1, k_2, k_3, k_4)(N_f) &= T_*^{IJKL}(k_1, k_2, k_3, k_4) \\
&+ \Theta_{IM}^K(N_f, N_*) \Theta_{JM}^L(N_f, N_*) (P_*(k_1)P_*(k_2)P_*(k_{13}) + 11 \text{ perms.}) \\
&+ \Xi_{IJK}^L(N_f, N_*) (P_*(k_1)P_*(k_2)P_*(k_3) + 3 \text{ perms.}), \tag{55}
\end{aligned}$$

As in the case of the bispectrum, the leading contribution to T_*^{IJKL} can be evaluated as [14]

$$\begin{aligned}
T_*^{IJKL}(k_1, k_2, k_3, k_4) &= -2V_{IJKL} \\
&\times \text{Re} \left[i g_{k_1}(\eta_*) g_{k_2}(\eta_*) g_{k_3}(\eta_*) g_{k_4}(\eta_*) \int_{-\infty}^{\eta_*} d\eta a^4(\eta) g_{k_1}^*(\eta) g_{k_2}^*(\eta) g_{k_3}^*(\eta) g_{k_4}^*(\eta) \right]. \tag{56}
\end{aligned}$$

For $k_i \eta_* \ll 1$, Eq. (56) reduces to

$$\begin{aligned}
T_*^{IJKL}(k_1, k_2, k_3, k_4) &= -\frac{H^4 k_t^3}{8(k_1 k_2 k_3 k_4)^3} V_{IJKL} \\
&\times \left[\left(\frac{1}{2} \log(k_t \eta_*)^2 + \gamma \right) \left(-\frac{1}{3} + \frac{\sum_{i < j} k_i k_j}{k_t^2} - \frac{\sum_{i < j < \ell} k_i k_j k_\ell}{k_t^3} \right) \right. \\
&\left. + \frac{4}{9} - \frac{\sum_{i < j} k_i k_j}{k_t^2} + \frac{k_1 k_2 k_3 k_4}{k_t^4} \right], \tag{57}
\end{aligned}$$

where $k_t = k_1 + k_2 + k_3 + k_4$. T_* is typically smaller than the other terms on the right hand side of Eq. (55), which corresponds to the super-horizon evolution of the trispectrum due to the fourth order interactions. Hence, as in the case of the bispectrum, the leading trispectrum of $\delta\phi^I$ at the end of inflation comes from super-horizon evolution,

$$\begin{aligned}
T^{IJKL}(k_1, k_2, k_3, k_4)(N_f) &= \Theta_{IM}^K(N_f, N_*) \Theta_{JM}^L(N_f, N_*) (P_*(k_1)P_*(k_2)P_*(k_{13}) + 11 \text{ perms.}) \\
&+ \Xi_{IJK}^L(N_f, N_*) (P_*(k_1)P_*(k_2)P_*(k_3) + 3 \text{ perms.}), \tag{58}
\end{aligned}$$

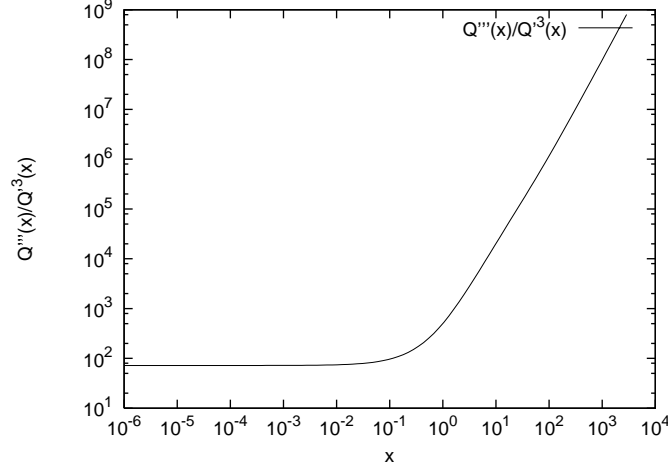


FIG. 3: Plot of $Q'''(x)/Q'^3(x)$.

Therefore, the corresponding non-linear parameters τ_{NL} and g_{NL} are given by

$$\tau_{NL} = \frac{1}{(xQ'(x))^2} \frac{\Gamma^2 \tilde{\Theta}^I \tilde{\Theta}_I}{\Gamma_K \Gamma^K} + \frac{2\Gamma^2 \tilde{\Gamma}_{(2)}^I \tilde{\Theta}_I}{(xQ'(x))^2 (\Gamma_K \Gamma^K)^{3/2}} + \frac{2x^2 Q''(x) \Gamma \Gamma_I \tilde{\Theta}^I}{(xQ'(x))^3 \Gamma_K \Gamma^K} \\ + \frac{\Gamma^2 \tilde{\Gamma}_{(2)}^I \tilde{\Gamma}_{I(2)}}{(xQ'(x))^2 (\Gamma_L \Gamma^L)^2} + \frac{2x^2 Q''(x)}{(xQ'(x))^3} \frac{\Gamma \Gamma_I \tilde{\Gamma}_{(2)}^I}{(\Gamma_K \Gamma^K)^{3/2}} + \frac{Q''^2(x)}{Q'^4(x)}, \quad (59)$$

$$\frac{54}{25} g_{NL} = \frac{\Gamma^2 \tilde{\Xi}}{(xQ'(x))^2 \Gamma_I \Gamma^I} + \frac{1}{(xQ'(x))^2} \frac{\Gamma^2 \tilde{\Gamma}_{(2)}^I \tilde{\Theta}_I}{(\Gamma_J \Gamma^J)^{3/2}} + \frac{x^2 Q''(x)}{(xQ'(x))^3} \frac{\Gamma \Gamma_I \tilde{\Theta}^I}{\Gamma_K \Gamma^K} \\ + \frac{1}{(xQ'(x))^2} \frac{\Gamma^2 \Gamma_I \tilde{\Gamma}_{(3)}^I}{(\Gamma_J \Gamma^J)^2} + \frac{3x^2 Q''(x)}{(xQ'(x))^3} \frac{\Gamma \tilde{\Gamma}_{(2)}^I \Gamma_I}{(\Gamma_J \Gamma^J)^{3/2}} + \frac{Q'''(x)}{Q'^3(x)}, \quad (60)$$

where

$$\tilde{\Xi} = \frac{\Gamma_I \Gamma_J \Gamma_K \Gamma_L}{(\Gamma_M \Gamma^M)^2} \Xi^{IJKL}, \quad \tilde{\Gamma}_{(3)}^I = \frac{\Gamma_J \Gamma_K \Gamma^{IJK}}{\Gamma_L \Gamma^L}. \quad (61)$$

As in the case of the bispectrum, the last term of g_{NL} represents the contribution from the non-linear relation between ζ and $\delta\Gamma$ and depends on Γ only through the argument x of $Q(x)$. $Q'''(x)/Q'^3(x)$ takes minimum value 72 at $x = 0$, and larger values for $x > 1$ (see Fig. 3). We see that all three non-linear parameters, f_{NL} , τ_{NL} and g_{NL} , can be written only in terms of the vector quantities Γ^I , $\tilde{\Gamma}_{(2)}^I$, $\tilde{\Gamma}_{(3)}^I$ and $\tilde{\Theta}^I$.

If Γ^{IJK} and the intrinsic non-Gaussianity of scalar field fluctuations are negligibly small, we have the following relation

$$g_{NL} = \frac{5}{3} \frac{Q''(x)}{Q'^2(x)} f_{NL} + \frac{25}{54} \frac{Q'''(x)}{Q'^3(x)}. \quad (62)$$

In particular, for $x \ll 1$, this relation reduces to

$$g_{NL} = 10 f_{NL} - \frac{50}{3}. \quad (63)$$

Hence g_{NL} has the same order of magnitude as f_{NL} for $x \ll 1$ and becomes much larger than f_{NL} for $x \gg 1$. This is in contrast with the case of standard single-field inflation, where g_{NL} is to second order in the slow-roll parameters. Notice that the relation Eq. (62) depends on Γ only through the argument $x = \Gamma/H_0$ and is independent of the functional form of $\Gamma(\phi)$ except for conditions on Γ^{IJK} .

V. CONCLUSIONS

The modulated reheating scenario generates a primordial curvature perturbation due to the spatial fluctuations of the inflaton decay rate. Such a scenario induces larger non-Gaussianity in the perturbation than that of the simple inflation scenario. Future observations such as the Planck satellite [18] may be able to detect or constrain the level of the non-Gaussianity, and thereby distinguish the different scenarios.

We have given expressions for the power spectrum, bispectrum and trispectrum to leading order in the modulated reheating scenario, allowing for multi-field dependence on the inflaton decay rate. The leading contribution to the bispectrum and trispectrum comes from the super-horizon evolution. If the intrinsic non-Gaussianity of the scalar fields and third derivative of the decay rate Γ^{IJK} are subdominant, then we have a simple relation between two non-linear parameters f_{NL} and g_{NL} , which is independent of the detailed form of the decay rate except for the condition on Γ^{IJK} . Hence, g_{NL} has at least the same order of magnitude as f_{NL} .

We have also given a general inequality between the bispectrum and the trispectrum $\tau_{NL} \geq \frac{36}{25} f_{NL}^2$ which is true for other inflationary scenarios as long as the non-Gaussianity comes from the super-horizon evolution. This inequality allows f_{NL} to be vanishingly small while τ_{NL} remains finite. In such a case, the leading non-Gaussianity comes not from the bispectrum but from the trispectrum.

Acknowledgments

We would like to thank Matias Zaldarriaga for useful comments. We are also grateful to W. Kelly for correcting the English. M.Y. is supported in part by JSPS Grant-in-Aid for Scientific Research Nos. 18740157 and 19340054.

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